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Top Quark Decays into Heavy Quark Mesons

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Abstract

For top quark decays into heavy quark mesons Υ and $\bar{B}_c^*$, a complete calculation to the leading order both in QCD coupling constant α_s and in v, the typical velocity of the heavy quarks inside the mesons, is performed. Relatons between the top quark mass and the decay branching ratios are studied. Comparion with the results which are obtained by using the quark frangmentation functions is also discussed. The branching ratios are consistent (within a factor of $2 \sim 3$) with that obtained using fragmentation functions at $m_t \sim 150$ GeV.

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I Introduction

The success of Standard Model (SM)[1][2][3] suggests that the top quark must exit [4]. Recently, from the direct search at the Tevatron, the **CDF** and **D0** groups confirmed the existence of heavy top quark[5][6], which has a mass of $(176 \pm 8 \pm 10)$ GeV or $(199^{+19}_{-21} \pm 22)$ GeV. Then next the experimental studies will focus on the determination of its properties. Among them, the precision measurement of the top quark mass and of the production cross sections and distributions will certainly be in the first studies of interest. Since the fermion mass generation can be closely related to the electroweak symmetry breaking, one expects to find some residual effects of the breaking in accordance with the generated mass especially in the much heavier top quark sector. Therefore, it is important to study the top quark system as a direct tool to probe new physics effects[4].

As the new found top quark mass is much heavier than that one previously expected, its decay widths in many processes should be reevaluated. With future upgrade and increases in total integrated luminosity, experiments at the Fermilab Tevatron will carry a carefull investigation of the production and decay of top quark. With the operation of CERN Large Hadron Collider (LHC), one expects to obtain roughly $10^7 - 10^8$ $t\bar{t}$ pairs per year [7] and then one may be able to observe various top decay channels, which will give further tests of the SM and heavy meson production mechamisms. On the other hand, the study of heavy quarkonium and heavy meson production in turn provides a ground to precisely test the perturbative quantum chromodynamics (PQCD).

Within past few years, much progress has been made in the study of production mechanism of heavy quarkonium and heavy mesons with large transverse momentum at high energies. The parton fragmentation in the heavy quarkonium and heavy meson production has been carefully studied [8][9] and the universal fragmentation functions for different spin and angular momentum states have been obtained. With these functions, the branching ratios of heavy quarkonium and heavy meson production for various channels may be easily obtained if the fragmentation mechanism is in dominance. Based on the new factorization formulas[10] and the production mechanism, one may explain some new experiment results[11]. Hence, in what energy region and in what degree of precision the fragmentation mechanism works in a specific process should be concerned.

The most important concequence of a heavy top quark is that, because its lifetime is short, after created in free state it does not have time to bind with light quarks to form a

bound state[13]. Therefore, the final frontier for the study of heavy-quark-antiquark bound states may be the B_c and $\bar{B}_c$, which carry flavors explicitly and are the ground states of bound $\bar{b}c$ and $c\bar{b}$ systems. The properties of these mesons and the possibility to find them at existing accelerators have been discussed in Refs.[8][14]. The top rare decay to $\bar{B}_c^*$ is also an important channel to study the still undiscovered heavy quark-antiquark bound state at LHC in the near future, though it is not practical at present with limited top quark events.

In this paper we present the full leading order calculation of top quark decays into $\bar{B}_c^*$ and Υ in strong coupling constant α_s as well as in v^2, where v is the typical relative velocity of the heavy quarks inside the bound state. The reliance of the branching ratios of heavy quarkonium and heavy meson production in top decay on the mass of top quark is also studied.

II Formalism

The amplitudes for $t \to W^+ c\bar{B}_c^*$ and $t \to W^+ b\Upsilon$ involve two Feynman diagrams respectively as shown in Fig.1. They can be writen down using standard Feynman rules for bound state productions and decays[12][19] or from the description of Bethe-Salpeter equation for bound states. In the following we will start with discussion from the latter concept.

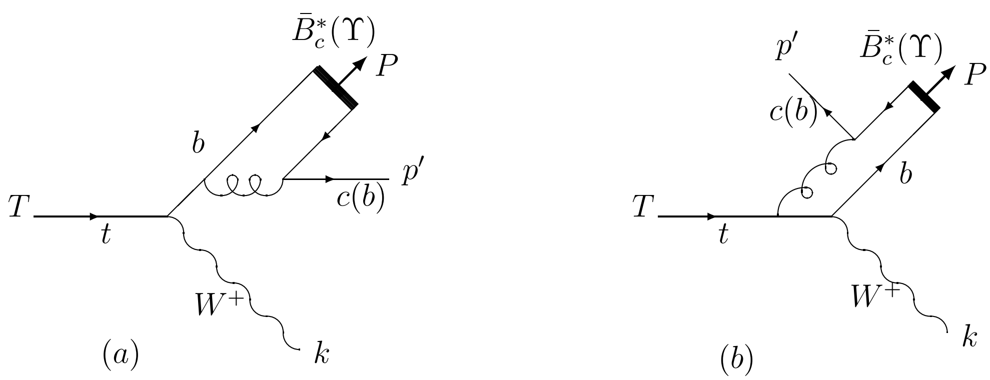

Fig.1

Based on the Mandelstam formalism [16] the amplitude of top decays corresponding to

diagrams Fig.1 is written as

$$M=\frac{4gg_s^2}{3\sqrt{6}}\bar{u}(q_2,\bar{s})\int\frac{d^4q}{(2\pi)^4}\left\{\mathcal{A}_1+\mathcal{A}_2\right\}v(q_1,s). \tag{1}$$

Here $s,\bar{s}$ are the spin projections of the quark and the antiquark; q_1 and q_2 are their momenta; g_s is the coupling constant of QCD; g is the weak coupling constant in WS model.

The general form of the partial width for the top decays under consideraton is

$$\Gamma(t\to threebody)=\frac{1}{256\pi^3m_t^3}\int_{s_2^-}^{s_2^+}ds_2\int_{s_1^-}^{s_1^+}ds_1\overline{\sum}|M^2|. \tag{2}$$

In the following we will discuss the two channels of top quark decaying into vector mesons $\bar{B}_c^*$ and Υ which might be in the first place being measuerd in the future experiment on top rare decay.

A. $t\to\bar{B}_c^*+W^++c$

At leading order in α_s, the corresponding $\mathcal{A}_1,\mathcal{A}_2$ in Eqn.1 for $t\to\bar{B}_c^*+W^++c$ are

$$\mathcal{A}_1=\gamma_\mu\frac{\chi(q)}{(p'+p_2)^2}\gamma^\mu\frac{\not{p}+\not{p}'+m_b}{(p'+p_2)^2-m_b^2}\not{\epsilon}_wP_L, \tag{3}$$

$$\mathcal{A}_2=\gamma_\mu\frac{\chi(q)}{(p'+p_2)^2}\not{\epsilon}_wP_L\frac{\not{p}_1+\not{k}+m_b}{(p_1+k)^2-m_t^2}\gamma^\mu. \tag{4}$$

Where the $P_L=\frac{1}{2}(1-\gamma_5)$; $\chi(q)$ is the Bethe-Salpeter(BS) wave function of the$\bar{B}_c^*$ meson with inner relative momentum q between the heavy quarks; ϵ_w is the polarization vector of W^+ boson. In Eqn.(3)(4), p,k,p',p_1 and p_2 are the momenta of $\bar{B}_c^*$ meson, W^+ boson, c quark, b quark, and $\bar{c}$ quark, respectively. For $\bar{B}_c^*$ meson, the two quarks have different masses inside the bound state. Their momenta p_1 and p_2 have the relation

$$p_1=\eta_1p+q\ ,\quad p_2=\eta_2p-q, \tag{5}$$

where $\eta_1=\frac{m_b}{(m_b+m_c)}$ and $\eta_2=\frac{m_c}{(m_b+m_c)}$. Under the instantaneous approximation[15] with the negative energy projectors being neglected, the BS wave function $\chi(q)$ of the bound state may be expressed as

$$\chi(q)=\frac{i}{2\pi}\frac{M-E_1-E_2}{(P_{10}-E_1)(P_{20}-E_2)}\Phi(\vec{q}). \tag{6}$$

Here M is the mass of bound state; P_{10} and P_{20} are the time components of quark and antiquark momenta inside the meson, and E_1, E_2 are their energies. From the standard BS wave functions in the approximation that the negative energy projectors are omitted, the vector meson wave function can be projected out as :

$$\Phi(\vec{q}) = \frac{1}{M}\sum_{sm}\langle JM|1sLm\rangle\Lambda_+^1(\vec{q})\gamma_0 \not\epsilon(M+ \not P)\gamma_0\Lambda_-^2(-\vec{q})\psi_{Lm}(\vec{q}), \tag{7}$$

where ϵ is the polarization vector associated with the spin triplet states. $\Lambda_+^1(\vec{q})$ and $\Lambda_-^2(-\vec{q})$ are positive energy projection operators of both quark and antiquark .

$$\Lambda_+^1(\vec{q}) = \frac{E+\gamma_0\vec{\gamma}\cdot\vec{q}+m_c\gamma_0}{2E}, \quad \Lambda_-^2(-\vec{q}) = \frac{E+\gamma_0\vec{\gamma}\cdot\vec{q}-m_c\gamma_0}{2E}. \tag{8}$$

After taking the nonrelativistic approximation the bound state wave function may be reduced to a simple form. As in Refs.[18][19], we also ignore the dependence of $\mathcal{A}_1$ and $\mathcal{A}_2$ on the relative momentum q. It may be considered as the lowest order approximation. After considering the normalization of $\Phi(\vec{q})$, we can get

$$\int \chi(q)\frac{d^4q}{(2\pi)^4} = \frac{1}{4\sqrt{M_{\bar{B}_c^*}\pi}} \not\epsilon(\not P + M_{\bar{B}_c^*})R_{\bar{B}_c^*}(0). \tag{9}$$

Here $R_{\bar{B}_c^*}(0)$ is the radiual wave function at origin. Hence the matrix element squared can be carried out directly,

$$\overline{\sum}|M|^2 = \overline{\sum}(|M_1|^2+|M_2|^2+2ReM_1M_2^*). \tag{10}$$

The square of the full amplitude is complicated and lengthy, it can be found in Appendix A. The kinematical variables entering Eq.(2) are defined as

$$s_1 = (p'+p)^2\ , \quad s_2 = (p+k)^2, \tag{11}$$

in which the phase space boundaries are readily found to be

$$s_1^{\pm} = m_c^2+M_{\bar{B}_c^*}^2-\frac{1}{2s_2}[(s_2-m_t^2+m_c^2)(s_2+M_{\bar{B}_c^*}^2-m_w^2)\mp\lambda^{\frac{1}{2}}(s_2,m_t^2,m_c^2)\lambda^{\frac{1}{2}}(s_2,M_{\bar{B}_c^*}^2,m_w^2)], \tag{12}$$

and

$$s_2^- = (M_{\bar{B}_c^*}+m_w)^2, \tag{13}$$

$$s_2^+ = (m_t-m_c)^2, \tag{14}$$

where

$$\lambda(x,y,z) = (x-y-z)^2 - 4yz. \tag{15}$$

B. $t \to \Upsilon + W^+ + b$

The $\mathcal{A}_1$ and $\mathcal{A}_2$ in Eq.(1) for $t \to W^+ b\Upsilon$ are the same form as Eqs.(3)(4) except $\chi(q)$ representing bottomonium BS wave function and $c(\bar{c})$ replaced by $b(\bar{b})$ quark. Under the same argument as in A, we have

$$\int \chi(q)\frac{d^4q}{(2\pi)^4} = \frac{1}{4\sqrt{M_\Upsilon \pi}} \not{\epsilon}(\not{P} + M_\Upsilon)R_\Upsilon(0). \tag{16}$$

The matrix squared is given in Appendix B. The phase-space bounderies to be used in the numerical evaluation of Eq.(2) in this channel are given by

$$s_1^{\pm} = m_b^2 + m_\Upsilon^2 - \frac{1}{2s_2}[(s_2 - m_t^2 + m_b^2)(s_2 + m_\Upsilon^2 - m_w^2) \mp \lambda^{\frac{1}{2}}(s_2, m_t^2, m_b^2)\lambda^{\frac{1}{2}}(s_2, m_\Upsilon^2, m_w^2)], \tag{17}$$

and

$$s_2^- = (m_\Upsilon + m_w)^2, \tag{18}$$

$$s_2^+ = (m_t - m_b)^2. \tag{19}$$

III Numerical Calculation and Results

We can now make use of this formalism to evaluate the decay rates of top quark to Υ and $\bar{B}_c^*$ mesons. In numerical calculations, we take the parameters as follows:

$$\begin{gathered} m_b = 4.9GeV, \quad m_c = 1.5GeV, \quad m_t = 176GeV, \quad m_w = 80.22GeV. \\ \alpha_s = 0.26 \quad for\ t \to W^+ c\bar{B}_c^*, \\ \alpha_s = 0.19 \quad for\ t \to w^+ b\Upsilon. \end{gathered} \tag{20}$$

Under the non-relativistic approximation,

$$M_\Upsilon = 2m_b, \quad M_{\bar{B}_c^*} = m_b + m_c. \tag{21}$$

The radial wave functions at the origin for $\bar{B}_c^*$ and Υ can be estimated through potential models[17] and electric decay rate. As in Ref.[9] we take

$$\begin{aligned} \left|R_{\bar{B}_c^*}(0)\right|^2 &= (1.18GeV)^3, \\ |R_\Upsilon(0)|^2 &= (1.8GeV)^3. \end{aligned} \tag{22}$$

Finally, we obtain the partial width of the decay $t \to \bar{B}_c^* + W^+ + c$:

$$\Gamma(t \to \bar{B}_c^* W^+ c) = 0.64MeV, \tag{23}$$

giving

$$\frac{\Gamma(t \to \bar{B}_c^* W^+ c)}{\Gamma(t \to W^+ b)} = 4.12 \times 10^{-4}. \tag{24}$$

The partial width of the decay $t \to \Upsilon + W^+ + b$ is

$$\Gamma(t \to \Upsilon + W^+ + b) = 1.54 \times 10^{-2} MeV, \tag{25}$$

giving

$$\frac{\Gamma(t \to \Upsilon W^+ b)}{\Gamma(t \to W^+ b)} = 0.98 \times 10^{-5}. \tag{26}$$

The complete leading order calculation of the decay rate for $t \to W^+ b \Upsilon$ has already been calculated[18] for top quark with a mass of 100 GeV and slightly different values for other parameters, giving the branching fraction 4×10^{-7}. In our calculation , we find the branching ratio is two order of magnitude greater when the top quark mass is set to 176 GeV. This indicates that the Υ and $\bar{B}_c^*$ meson productions in top decay is really a quark fragmentation process if the top quark is heavy enough. Using the fragmentation function obtained in Ref.[9], one can readily obtain the branching ratios for $t \to \bar{B}_c^* W^+ c$ and $t \to \Upsilon W^+ b$. However, in how much degree the univeresal fragmentation functions suit the top rare decays is not yet very clear. To answer this question, we give out the relationships between the top quark mass and the branching ratios in Fig.2 and Fig.3 respectively for $t \to \Upsilon W^+ b$ and $t \to \bar{B}_c^* W^+ c$. From Fig.2 and Fig.3 we can see that the branching ratios increase rapidly as the top quark mass increases when $m_t \leq 120$ GeV, and the heavier the top quark is, the less the branching ratios change. When top quark mass is heavier than 200 GeV, the branching ratios seem to be saturated and no longer depend on the top quark mass. That means the univeral fragmentation functions works well. As for the numerical results, with the same parameters as used in Refs.[9] at $m_t = 176$ GeV, our complete leading order calculations give $R(t \to \Upsilon W^+ b) = 0.98 \times 10^{-5}$, which is about two times smaller than in Ref.[9] using the universal fragmentation function and $R(t \to \bar{B}_c^* W^+ c) = 4.12 \times 10^{-4}$, which is almost as large as Ref.[9] using the universal fragmentation function.

In Fig.4 and Fig.5 we also show the relations of these branching ratios to the strong coupling constant α_s.

IV Conclusion

We have presented two dominant three-body decays of top quark to ground state heavy mesons. The decay widths and branching ratios are $0.64 MeV$ and 4.12×10^{-4} for $\bar{B}_c^*$ productionin ; $1.54 \times 10^{-2} MeV$ and 0.98×10^{-5} for Υ production . As about 10^8 $t\bar{t}$ pairs per year will be produced at the planned LHC, this means it is possible to accumulate about 10^4 $\bar{B}_c^*$ events and 10^3 Υ events a year. This will make it possible to discover the $\bar{B}_c^*$ meson and to further study its properties experimentally. At NLC (Next Linear Collider), rare decays of top quark could be searched for down to a very small branching ratos, and a systematic and general search for top rare decay modes may be possible. Then the two top decay modes discussed in this paper might correspond to a detectable level at NLC. On the other hand the results obtained in this paper will also be a helpful reference to other production mechamisms concerning the top quark decay.

Acknowlegement

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APPENDIX A

Following we give the expressions of squared matrix for $t \to W^+ c\bar{B}_c^*$. The vertex coefficents are not taken into consideration. In this appendix we take $\widetilde{M}_{\bar{B}_c^*} = \frac{1}{2}M_{\bar{B}_c^*}$ just for calculation convenience.

$$
\begin{aligned}
\overline{\sum}|M_1|^2 &= \frac{1}{(4m_b^2\widetilde{M}^2_{\bar{B}_c^*}m_w^2 + 8m_b\widetilde{M}^2_{\bar{B}_c^*}m_c m_w^2 + 4\widetilde{M}^2_{\bar{B}_c^*}m_c^2 m_w^2)} \\
&\times \frac{1}{(\eta_1 s_2 - 4\eta_2\widetilde{M}^2_{\bar{B}_c^*} + \eta_2 m_w^2 + 4\eta_2^2\widetilde{M}^2_{\bar{B}_c^*} - m_t^2)^2}\frac{1}{4\eta_2^2(s_1 - m_b^2)^2} \\
&\times \{64m_b^2\widetilde{M}^2_{\bar{B}_c^*}(k\cdot p)^2(p\cdot p')(t\cdot p) - 64m_b\widetilde{M}^2_{\bar{B}_c^*}m_t^2(m_b+m_c)(k\cdot p)^2(p\cdot p') \\
&+ 128m_b^2\widetilde{M}^3_{\bar{B}_c^*}m_c(k\cdot p)^2(t\cdot p) - 320m_b\widetilde{M}^3_{\bar{B}_c^*}m_c m_t^2(m_b+m_c)(k\cdot p)^2 \\
&+ 32(4m_b^2\widetilde{M}^2_{\bar{B}_c^*} + m_b^2m_t^2 + 4m_b^2m_w^2 + 2m_bm_cm_t^2 \\
&+ 8m_bm_cm_w^2 + m_c^2m_t^2 + 4m_c^2m_w^2)\widetilde{M}^2_{\bar{B}_c^*}(k\cdot p)^2(t\cdot p') \\
&- 64\widetilde{M}^2_{\bar{B}_c^*}(2m_b^2\widetilde{M}^2_{\bar{B}_c^*} - m_b^2m_t^2 - m_b^2m_w^2 - 2m_bm_cm_t^2 \\
&- 2m_bm_cm_w^2 - m_c^2m_t^2 - m_c^2m_w^2)(k\cdot p)(k\cdot p')(t\cdot p) \\
&+ 128m_b^2\widetilde{M}^4_{\bar{B}_c^*}(k\cdot p)(k\cdot t)(p\cdot p') + 64m_b\widetilde{M}^2_{\bar{B}_c^*}m_w^2(m_b+m_c)(k\cdot p)(p\cdot p')(t\cdot p) \\
&+ 64\widetilde{M}^3_{\bar{B}_c^*}(-4m_b^2m_c\widetilde{M}^2_{\bar{B}_c^*} + m_t^2m_b^2m_c + 4m_w^2m_b^2m_c + 2m_t^2m_c^2m_b \\
&+ 8m_w^2m_bm_c^2 + m_c^3m_t^2 + 4m_c^3m_w^2)(k\cdot p)(k\cdot t) \\
&- 160\widetilde{M}^2_{\bar{B}_c^*}m_t^2m_w^2(m_b+m_c)^2(k\cdot p)(p\cdot p') \\
&+ 192m_bm_c\widetilde{M}^3_{\bar{B}_c^*}m_w^2(m_b+m_c)(k\cdot p)(t\cdot p) \\
&+ 704m_b\widetilde{M}^4_{\bar{B}_c^*}m_w^2(m_b+m_c)(k\cdot p)(t\cdot p') \\
&- 672m_c\widetilde{M}^3_{\bar{B}_c^*}m_t^2m_w^2(m_c+m_b)^2(k\cdot p) \\
&- 64\widetilde{M}^4_{\bar{B}_c^*}(4m_b^2\widetilde{M}^2_{\bar{B}_c^*} + m_b^2m_t^2 - 2m_b^2m_w^2 + 2m_bm_cm_t^2 - 4m_bm_cm_t^2 \\
&+ m_c^2m_t^2 - 2m_c^2m_w^2)(k\cdot p')(k\cdot t) + 192m_b\widetilde{M}^4_{\bar{B}_c^*}m_w^2(m_b+m_c)(k\cdot p')(t\cdot p) \\
&- 128\widetilde{M}^4_{\bar{B}_c^*}m_t^2m_w^2(m_b+m_c)^2(k\cdot p') + 768m_bm_c\widetilde{M}^5_{\bar{B}_c^*}m_w^2(m_b+m_c)(k\cdot t) \\
&+ 192m_b\widetilde{M}^4_{\bar{B}_c^*}m_w^2(m_b+m_c)(k\cdot t)(p\cdot p') \\
&+ 16\widetilde{M}^2_{\bar{B}_c^*}m_w^2(8\widetilde{M}^2_{\bar{B}_c^*}m_b^2 + 2m_b^2m_t^2 - m_b^2m_w^2 \\
&+ 4m_bm_cm_t^2 - 2m_bm_cm_w^2 + 2m_c^2m_t^2 - m_c^2m_w^2)(p\cdot p')(t\cdot p) \\
&- 512m_b\widetilde{M}^4_{\bar{B}_c^*}m_t^2m_w^2(m_b+m_c)(p\cdot p') \\
&+ 64\widetilde{M}^3_{\bar{B}_c^*}m_cm_w^2(8m_b^2\widetilde{M}^2_{\bar{B}_c^*} + 2m_b^2m_t^2 - m_b^2m_w^2
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 4m_bm_cm_t^2-2m_bm_cm_w^2+2m_c^2m_t^2-m_c^2m_w^2)(t\cdot p)\\
&+ \quad 32\widetilde{M}_{\bar{B}_c^*}^4m_w^2(28m_b^2\widetilde{M}_{\bar{B}_c^*}^2+7m_b^2m_t^2\\
&- \quad 5m_b^2m_w^2+17m_bm_cm_t^2-10m_bm_cm_w^2+7m_c^2m_t^2-5m_c^2m_w^2)(t\cdot p')\\
&- \quad 1408m_bm_c\widetilde{M}_{\bar{B}_c^*}^5m_t^2m_w^2(m_b+m_c)\}
\end{aligned}
$$

$$
\begin{aligned}
\overline{\sum}|M_2|^2 \quad = \quad & \frac{1}{4\eta_2^2(s_1-m_b^2)^2}\frac{1}{(s_1-m_b^2)^2}\\
\times \quad & \{(8\widetilde{M}_{\bar{B}_c^*}^2-m_c^2+m_b^2)(p\cdot p')(t\cdot p)\\
+ \quad & 2\widetilde{M}_{\bar{B}_c^*}^2(-4\widetilde{M}_{\bar{B}_c^*}^2-8\widetilde{M}_{\bar{B}_c^*}m_b+8\widetilde{M}_{\bar{B}_c^*}m_c+m_c^2+m_b^2-6m_bm_c)(t\cdot p')\\
+ \quad & 4\widetilde{M}_{\bar{B}_c^*}^2(m_c^2-3m_bm_c)(t\cdot p)+8\widetilde{M}_{\bar{B}_c^*}^2(p\cdot p')(t\cdot p')+2(p\cdot p')^2(t\cdot p')\\
+ \quad & \frac{4\widetilde{M}_{\bar{B}_c^*}^2}{m_w^2}(-4\widetilde{M}_{\bar{B}_c^*}^2+m_c^2+m_b^2-6m_bm_c)(k\cdot t)(k\cdot p')\\
+ \quad & \frac{16\widetilde{M}_{\bar{B}_c^*}^2}{m_w^2}(k\cdot t)(k\cdot p')(p\cdot p')+\frac{2}{m_w^2}(8\widetilde{M}_{\bar{B}_c^*}^2+m_b^2-m_c^2)(k\cdot t)(k\cdot p)(p\cdot p')\\
+ \quad & \frac{8\widetilde{M}_{\bar{B}_c^*}^2}{m_w^2}(m_c^2-3m_bm_c)(k\cdot t)(k\cdot p)+\frac{4}{m_w^2}(k\cdot t)(k\cdot p')(p\cdot p')^2)\}
\end{aligned}
$$

$$
\begin{aligned}
\overline{\sum}2Re(M_1M_2^*) \quad = \quad & \frac{1}{\widetilde{M}_{\bar{B}_c^*}m_w^2(m_b+m_c)}\frac{1}{(s_1-m_b^2)}\frac{1}{4\eta_2^2(s1-m_b^2)^2}\\
\times \quad & \frac{1}{(\eta_1s_2-4\eta_2\widetilde{M}_{\bar{B}_c^*}^2+\eta_2m_w^2+4\eta_2^2\widetilde{M}_{\bar{B}_c^*}^2-m_t^2)}\\
\times \quad & \{8m_b\widetilde{M}_{\bar{B}_c^*}(k\cdot p)^2(p\cdot p')(t\cdot p')-8m_b\widetilde{M}_{\bar{B}_c^*}^2(m_b+2\widetilde{M}_{\bar{B}_c^*}-m_c)(k\cdot p)^2(t\cdot p')\\
+ \quad & 4\widetilde{M}_{\bar{B}_c^*}m_cm_t^2(m_b^2+4m_b\widetilde{M}_{\bar{B}_c^*}+4m_c\widetilde{M}_{\bar{B}_c^*}-m_c^2)(k\cdot p)^2\\
- \quad & 8m_b\widetilde{M}_{\bar{B}_c^*}(k\cdot p)(k\cdot p')(p\cdot p')(t\cdot p)-32(k\cdot p)(k\cdot p')(t\cdot p')m_b\widetilde{M}_{\bar{B}_c^*}^3\\
+ \quad & 8\widetilde{M}_{\bar{B}_c^*}m_t^2(m_b+m_c)(k\cdot p)(k\cdot p')(p\cdot p')\\
+ \quad & 8m_b\widetilde{M}_{\bar{B}_c^*}^2(m_b+2\widetilde{M}_{\bar{B}_c^*}-m_c)(k\cdot p)(k\cdot p')(t\cdot p)\\
- \quad & 16\widetilde{M}_{\bar{B}_c^*}^2m_t^2(m_b^2+m_b\widetilde{M}_{\bar{B}_c^*}+m_c\widetilde{M}_{\bar{B}_c^*}-m_c^2)(k\cdot p)(k\cdot p')\\
- \quad & 8m_b\widetilde{M}_{\bar{B}_c^*}(k\cdot p)(k\cdot t)(p\cdot p')^2+32m_bm_c\widetilde{M}_{\bar{B}_c^*}^3(m_b+\widetilde{M}_{\bar{B}_c^*})(k\cdot p)(k\cdot t)\\
- \quad & 8m_b\widetilde{M}_{\bar{B}_c^*}^2(m_b+6\widetilde{M}_{\bar{B}_c^*}+m_c)(k\cdot p)(k\cdot t)(p\cdot p')
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 4\widetilde{M}_{\bar{B}_c^*} m_w^2 (m_b + m_c)(k\cdot p)(p\cdot p')(t\cdot p') \\
&+ \quad 8\widetilde{M}_{\bar{B}_c^*} m_c m_w^2 (m_b^2 + 4m_b\widetilde{M}_{\bar{B}_c^*} + 4\widetilde{M}_{\bar{B}_c^*} m_c - m_c^2)(k\cdot p)(t\cdot p) \\
&- \quad 8\widetilde{M}^2_{\bar{B}_c^*} m_w^2 (3m_b^2 + 5m_b\widetilde{M}_{\bar{B}_c^*} + 2m_b m_c + 5\widetilde{M}_{\bar{B}_c^*} m_c - m_c^2)(k\cdot p)(t\cdot p') \\
&+ \quad 32 m_b \widetilde{M}^3_{\bar{B}_c^*} (k\cdot p')^2 (t\cdot p) - 32\widetilde{M}^3_{\bar{B}_c^*} m_t^2 (m_b + m_c)(k\cdot p')^2 \\
&+ \quad 64 m_b \widetilde{M}^4_{\bar{B}_c^*} (\widetilde{M}_{\bar{B}_c^*} + m_c)(k\cdot p')(k\cdot t) \\
&+ \quad 12\widetilde{M}_{\bar{B}_c^*} m_w^2 (m_b + m_c)(k\cdot p')(p\cdot p')(t\cdot p) \\
&+ \quad 8\widetilde{M}^2_{\bar{B}_c^*} m_w^2 (-m_b^2 + m_b\widetilde{M}_{\bar{B}_c^*} + 2m_b m_c + m_c\widetilde{M}_{\bar{B}_c^*} + 3m_c^2)(k\cdot p')(t\cdot p) \\
&- \quad 64\widetilde{M}^3_{\bar{B}_c^*} m_w^2 (m_b + m_c)(k\cdot p')(t\cdot p') - 4\widetilde{M}_{\bar{B}_c^*} m_w^2 (m_b + m_c)(k\cdot t)(p\cdot p')^2 \\
&+ \quad 8\widetilde{M}^2_{\bar{B}_c^*} m_w^2 (m_b^2 + m_b\widetilde{M}_{\bar{B}_c^*} + m_c\widetilde{M}_{\bar{B}_c^*} - m_c^2)(k\cdot t)(p\cdot p') \\
&- \quad 8\widetilde{M}^3_{\bar{B}_c^*} m_c m_w^2 (m_b^2 + 4m_b\widetilde{M}_{\bar{B}_c^*} - 2m_b m_c + 4\widetilde{M}_{\bar{B}_c^*} m_c - 3m_c^2)(k\cdot t) \\
&+ \quad 16 m_b \widetilde{M}_{\bar{B}_c^*} m_w^2 (p\cdot p')^2 (t\cdot p) - 8\widetilde{M}_{\bar{B}_c^*} m_t^2 m_w^2 (m_b + m_c)(p\cdot p')^2 \\
&+ \quad 16 m_b \widetilde{M}^2_{\bar{B}_c^*} m_w^2 (m_c + 2\widetilde{M}_{\bar{B}_c^*})(p\cdot p')(t\cdot p) \\
&- \quad 48 m_b \widetilde{M}^3_{\bar{B}_c^*} m_w^2 (p\cdot p')(t\cdot p') - 4\widetilde{M}^2_{\bar{B}_c^*} m_t^2 m_w^2 (-m_b^2 + 8m_b\widetilde{M}_{\bar{B}_c^*} \\
&+ \quad 8m_c\widetilde{M}_{\bar{B}_c^*} + m_c^2)(p\cdot p') - 32 m_b \widetilde{M}^4_{\bar{B}_c^*} m_w^2 (2m_b + 3\widetilde{M}_{\bar{B}_c^*})(t\cdot p') \\
&- \quad 8 m_b m_c \widetilde{M}^3_{\bar{B}_c^*} m_w^2 (m_b - 8\widetilde{M}_{\bar{B}_c^*} + m_c)(t\cdot p) \\
&+ \quad 16 m_c \widetilde{M}^3_{\bar{B}_c^*} m_t^2 m_w^2 (2m_b^2 - m_b\widetilde{M}_{\bar{B}_c^*} + 2m_b m_c - m_c\widetilde{M}_{\bar{B}_c^*})\}
\end{aligned}
$$

In the above

$$
\begin{aligned}
k\cdot p \equiv p\cdot k = \frac{1}{2}(s_2 - 4\widetilde{M}^2_{\bar{B}_c^*} - m_w^2), \qquad & p\cdot p' = \frac{1}{2}(s_1 - m_c^2 - 4\widetilde{M}^2_{\bar{B}_c^*}) \\
p\cdot t \equiv t\cdot p = p\cdot p' + p\cdot k + 4\widetilde{M}^2_{\bar{B}_c^*}, \qquad & k\cdot t \equiv t\cdot k = -\frac{1}{2}(s_1 - m_t^2 - m_w^2) \\
k\cdot p' = k\cdot t - k\cdot p - m_w^2, \qquad & t\cdot p' = -\frac{1}{2}(s_2 - m_t^2 - m_c^2)
\end{aligned}
$$

APPENDIX B

In this appendix we give the expressions of squared matrix for $t \to W^+ + b + \Upsilon$. The vertex coefficients are also neglected.

$$
\begin{aligned}
\overline{\sum}|M_1|^2 &= \frac{1}{(s_2+m_w^2-2(m_b^2+m_t^2))^2}\frac{1}{(s_1-2m_b^2+m_b^2)^2} \\
&\times \{\frac{4}{m_w^2}(k\cdot p)^2(p\cdot p')(t\cdot p)-\frac{8m_t^2}{m_w^2}(k\cdot p)^2(p\cdot p')+\frac{8m_b^2}{m_w^2}(k\cdot p)^2(t\cdot p) \\
&+ \frac{8}{m_w^2}(m_b^2+m_t^2+4m_w^2)(k\cdot p)^2(t\cdot p')-\frac{40}{m_w^2}m_t^2m_b^2(k\cdot p)^2 \\
&+ \frac{8}{m_w^2}(-m_b^2+2m_t^2+2m_w^2)(k\cdot p)(k\cdot p')(t\cdot p)+\frac{8m_b^2}{m_w^2}(k\cdot p)(t\cdot k)(p\cdot p') \\
&+ \frac{16m_b^2}{m_w^2}(-m_b^2+m_t^2+4m_w^2)(k\cdot p)(t\cdot k)+8(k\cdot p)(p\cdot p')(t\cdot p) \\
&- 40m_t^2(k\cdot p)(p\cdot p')+24m_b^2(k\cdot p)(t\cdot p)+88m_b^2(k\cdot p)(t\cdot p') \\
&- 168m_b^2m_t^2(k\cdot p)-\frac{16m_b^2}{m_w^2}(m_b^2+m_t^2-2m_w^2)(k\cdot p')(t\cdot k) \\
&+ 24m_b^2(k\cdot p')(t\cdot p)-32m_b^2m_t^2(k\cdot p')+24m_b^2(t\cdot k)(p\cdot p') \\
&+ 96m_b^4(t\cdot k)+4(2m_b^2+2m_t^2-m_w^2)(p\cdot p')(t\cdot p)-64m_b^2m_t^2(p\cdot p') \\
&+ 16m_b^2(2m_b^2+2m_t^2-m_w^2)(t\cdot p)+8m_b^2(7m_b^2+7m_t^2-5m_w^2)(t\cdot p')-176m_b^4m_t^2\}
\end{aligned}
$$

$$
\begin{aligned}
\overline{\sum}|M_2|^2 &= \frac{1}{4(s_1-m_b^2)^4} \\
&\times \{8m_b^2(p\cdot p')(t\cdot p)-16m_b^4(t\cdot p')-8m_b^4(t\cdot p) \\
&+ 8m_b^2(p\cdot p')(t\cdot p')+2(p\cdot p')^2(t\cdot p') \\
&- \frac{32m_b^4}{m_w^2}(k\cdot t)(k\cdot p')-\frac{16m_b^4}{m_w^2}(k\cdot t)(k\cdot p) \\
&+ \frac{16m_b^2}{m_w^2}(k\cdot t)(k\cdot p')(p\cdot p') \\
&+ \frac{16m_b^2}{m_w^2}(k\cdot t)(k\cdot p)(p\cdot p')+\frac{4}{m_w^2}(k\cdot t)(k\cdot p')(p\cdot p')^2\}
\end{aligned}
$$

$$
\begin{aligned}
\overline{\sum}2Re(M_1M_2^*) &= \frac{1}{(s_1-m_b^2)^3}\frac{1}{(s_2+m_w^2-2(m_b^2+m_t^2))} \\
&\times \{\frac{2}{m_w^2}(k\cdot p)^2(p\cdot p')(t\cdot p')-\frac{4m_b^2}{m_w^2}(k\cdot p)^2(t\cdot p')+\frac{8m_t^2m_b^2}{m_w^2}(k\cdot p)^2 \\
&- \frac{2}{m_w^2}(k\cdot p)(k\cdot p')(p\cdot p')(t\cdot p)+\frac{4m_t^2}{m_w^2}(k\cdot p)(k\cdot p')(p\cdot p')
\end{aligned}
$$

$$
\begin{aligned}
+ &\quad \frac{4m_b^2}{m_w^2}(k\cdot p)(k\cdot p')(t\cdot p) - \frac{8m_b^2}{m_w^2}(k\cdot p)(k\cdot p')(t\cdot p') \\
- &\quad \frac{8m_t^2 m_b^2}{m_w^2}(k\cdot p)(k\cdot p') - \frac{2}{m_w^2}(k\cdot p)(k\cdot t)(p\cdot p')^2 \\
- &\quad \frac{16m_b^2}{m_w^2}(k\cdot p)(k\cdot t)(p\cdot p') + \frac{16m_b^4}{m_w^2}(k\cdot p)(k\cdot t) \\
+ &\quad 2(k\cdot p)(p\cdot p')(t\cdot p') + 16m_b^2(k\cdot p)(t\cdot p) - 28m_b^2(k\cdot p)(t\cdot p') \\
+ &\quad \frac{8m_b^2}{m_w^2}(k\cdot p')^2(t\cdot p) - \frac{16m_t^2 m_b^2}{m_w^2}(k\cdot p')^2 + \frac{32m_b^4}{m_w^2}(k\cdot p')(k\cdot t) \\
+ &\quad 6(k\cdot p')(p\cdot p')(t\cdot p) + 12m_b^2(k\cdot p')(t\cdot p) - 32m_b^2(k\cdot p')(t\cdot p') \\
- &\quad 2(k\cdot t)(p\cdot p')^2 + 4m_b^2(k\cdot t)(p\cdot p') - 8m_b^4(k\cdot t) \\
+ &\quad 4(p\cdot p')^2(t\cdot p) - 4m_t^2(p\cdot p')^2 + 12m_b^2(p\cdot p')(t\cdot p) \\
- &\quad 12m_b^2(p\cdot p')(t\cdot p') - 16m_b^2 m_t^2(p\cdot p') + 12m_b^4(t\cdot p) \\
- &\quad 40m_b^4(t\cdot p') + 8m_b^4 m_t^2\}
\end{aligned}
$$

Here

$$
\begin{aligned}
k\cdot p \equiv p\cdot k = \frac{1}{2}(s_2 - 4m_b^2 - m_w^2), &\qquad p\cdot p' = \frac{1}{2}(s_1 - 5m_b^2) \\
p\cdot t \equiv t\cdot p = p\cdot p' + p\cdot k + 4m_b^2, &\qquad k\cdot t \equiv t\cdot k = -\frac{1}{2}(s_1 - m_t^2 - m_w^2) \\
k\cdot p' = k\cdot t - k\cdot p - m_w^2, &\qquad t\cdot p' = -\frac{1}{2}(s_2 - m_t^2 - m_b^2)
\end{aligned}
$$

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Figure Captions

The Feynman diagrams for $t \to \bar{B}_c^* W^+ c$ and $t \to \Upsilon W^+ b$ process at leading order in α_s.

Fig.2 The branching ratio $R \equiv \Gamma(t \to \Upsilon + W^+ + b)/\Gamma(t \to W^+ b)$ versus $m_t(GeV)$

Fig.3 The branching ratio $R \equiv \Gamma(t \to \bar{B}_c^* + W^+ + c)/\Gamma(t \to W^+ b)$ versus $m_t(GeV)$

Fig.4 The branching ratio $R \equiv \Gamma(t \to \Upsilon + W^+ + b)/\Gamma(t \to W^+ b)$ versus α_s

Fig.5 The branching ratio $R \equiv \Gamma(t \to \bar{B}_c^* + W^+ + c)/\Gamma(t \to W^+ b)$ versus α_s

Fig. 2

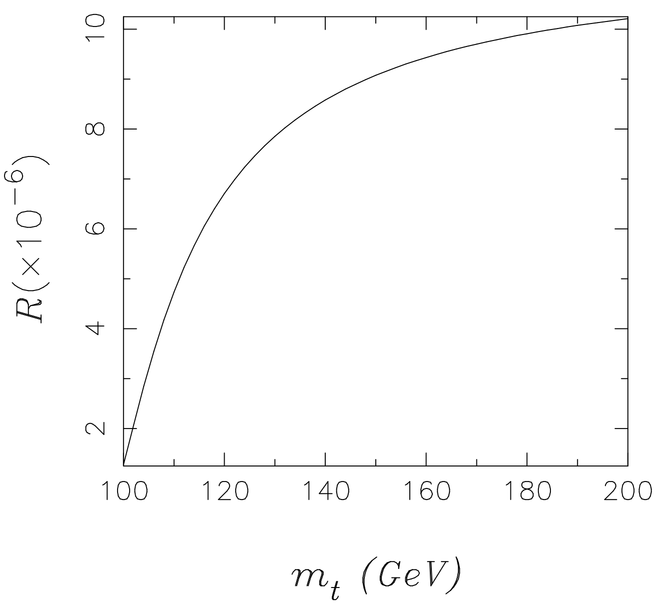

Fig. 3

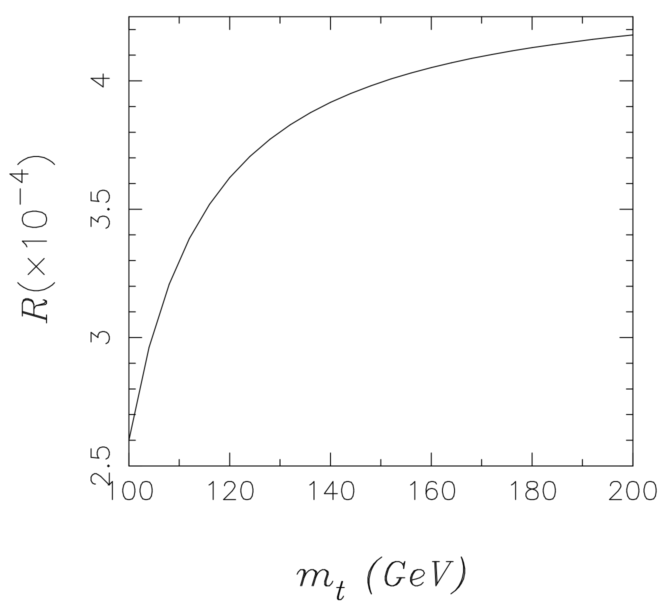

Fig. 4

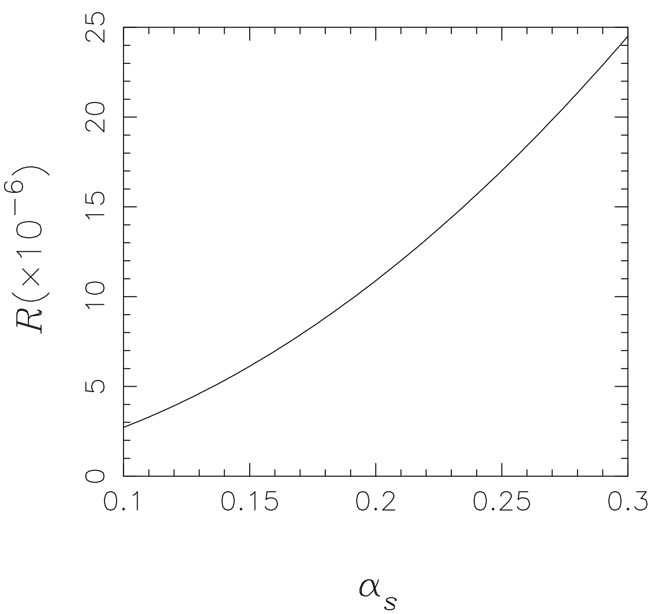

Fig. 5